



1.6.7 Switching between representations

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Outline

- In this topic, we will
 - See how to convert between the representations
 - Rectangular and polar
 - Learn about the complex exponential

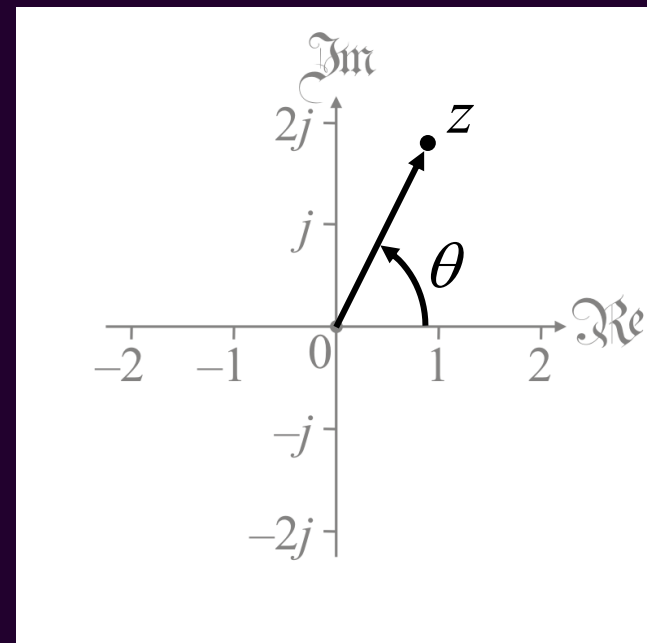




Rectangular to polar

- Given the complex number $z = \alpha + \beta j$,
this is in the rectangular representation
- In order to switch to the polar representation,
we must:

- Calculate $|z| = \sqrt{\alpha^2 + \beta^2}$
- Calculate $\arg(z)$
 - We have seen how to do this
 - You will only be asked to do
this where it is obvious:
 - Multiples of $\pi/4$ or 45°
- Otherwise, you would use:
`numpy.arctan2( beta, alpha )`
`std::atan( beta, alpha )`

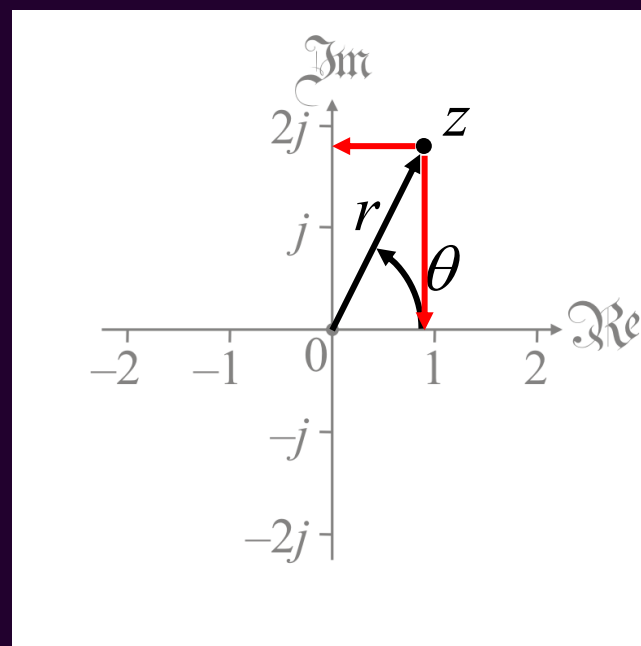




Polar to rectangular

- Given the polar representation $z = r \angle \theta$, to calculate the rectangular representation is even easier:

$$z = r \angle \theta = r \cos(\theta) + r \sin(\theta) j$$





Polar to rectangular

- For example, if $z = 5 \angle 60^\circ$,

$$\begin{aligned} z &= 5 \cos(60^\circ) + 5 \sin(60^\circ) j \\ &= 5 \frac{1}{2} + 5 \frac{\sqrt{3}}{2} j \\ &\approx 2.5 + 4.33 j \end{aligned}$$





Polar to rectangular

- Now, one observation:
 - In your calculus course, you will learn that

$$e^{j\theta} = \cos(\theta) + \sin(\theta)j$$

- Therefore,

$$\begin{aligned} re^{j\theta} &= r(\cos(\theta) + \sin(\theta)j) \\ &= r\cos(\theta) + r\sin(\theta)j = r\angle\theta \end{aligned}$$

- Therefore, the following two are equivalent:

$$re^{j\theta} = r\angle\theta$$





Summary

- In this topic, you now
 - Know how to convert between the rectangular and polar representations
 - Have been introduced to the complex exponential

$$e^{j\theta} = \cos(\theta) + \sin(\theta)j$$





References

- [1] https://en.wikipedia.org/wiki/Polar_coordinate_system





Switching between representations



Acknowledgments

None so far.





Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas. Mathematical equations are prepared in MathType by Design Science, Inc. Examples may be formulated and checked using Maple by Maplesoft, Inc.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

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